Math 412, Fall 2023 – Homework 2

Due: Wednesday, September 6th, at 9:00AM via Gradescope

Instructions: Students taking the course for three credit hours (undergraduates, most graduate students) should choose four of the following five problems to solve and turn in–if you do all five, only the first four will be graded. Graduate students taking the course for four credits should solve all five. Problems that use the word “describe”, “determine”, “show”, or “prove” require proof for all claims.

1. Recall that K_n denotes the complete graph on n vertices. Prove or disprove.

   a. For every $e, f \in E(K_n)$, $(K_n \setminus e) \cong (K_n \setminus f)$.
   b. For every $e_1, e_2, f_1, f_2 \in E(K_n)$ such that $e_1 \neq e_2$ and $f_1 \neq f_2$,
   $$((K_n \setminus e_1) \setminus e_2) \cong ((K_n \setminus f_1) \setminus f_2).$$

2. Let G be a graph, let e be an edge of G and let W be a closed walk in G such that e is in W an odd number of times. Prove that W contains a cycle that contains the edge e.

3. Determine for which values m and n the complete bipartite graph $K_{m,n}$ has an Eulerian circuit.

4. Prove that a loopless graph G is bipartite if and only if every subgraph H of G has an independent set of size at least $|V(H)|/2$.

5. Two Eulerian circuits in a graph G are *equivalent* if they have the same cyclic sequence of edges or if one cyclic sequence is the reverse of the other. For example,
   $$(v_0, e_1, v_1, e_2, v_3, e_3, v_0) \quad \text{and} \quad (v_1, e_2, v_3, e_3, v_0, e_1, v_1) \quad \text{and} \quad (v_0, e_3, v_3, e_2, v_1, e_1, v_0)$$
   are all equivalent Eulerian circuits. How many equivalence classes of Eulerian circuits are there for the following graph G? This problem does *not* require proof; however, you *do* need to write down an Eulerian circuit for each equivalence class.